Adaptive Anchor Weighting for Improved Localization with Levenberg-Marquardt Optimization

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***Abstract*—This paper introduces an iterative and weighted localization method that utilizes a unique cost function formulation to significantly enhance the performance of positioning systems. The system employs locators, such as Gateways (GWs), to estimate and track the position of an End Node (EN). Performance is evaluated relative to the number of locators, with known locations determined through calibration. Performance evaluation is presented utilizing low cost single-antenna Bluetooth Low Energy (BLE) devices. The proposed approach can be applied to alternative Internet of Things (IoT) modulation schemes, as well as Ultra WideBand (UWB) or millimeter-wave (mmWave) based devices. In non-line-of-sight (NLOS) scenarios, using four or eight locators yields a 95th percentile localization performance of 2.2 meters and 1.5 meters, respectively, in a 4,305 square feet indoor area with BLE 5.1 devices. This method outperforms conventional RSSI-based techniques, achieving a 51% improvement with four locators and a 52 % improvement with eight locators. Future work involves modeling interference impact and implementing data curation across multiple channels to mitigate such effects.**

***Index Terms*—lateration, least squares, Levenberg-Marquardt algorithm, localization, path-loss, rms error, RSSI, sensors, shadow fading, weighted localization**

I. INTRODUCTION

In the literature, a beacon is defined as a locator or an anchor node with a pre-calibrated and known position, serving as a stationary reference point in indoor positioning systems [1], [2], [3], [4], [5], [6]. It transmits or receive signals that the system leverages to ascertain a device's position relative to the beacon's coordinates. This paper explores the utilization of cost-effective Bluetooth Low Energy (BLE) technology, presuming a single antenna per device for determining device location by leveraging other BLE-enabled locators. The Received Signal Strength Indicator (RSSI) is employed for estimating the initial distances by utilizing pre-calibrated path-loss measurements obtained at a specific indoor location [7]. The proposed method does not rely merely on RSSI based methods and can use any technology for initial distance estimation. The main focus of the proposed method is to improve localization accuracy, given a set of distance estimates to multiple anchors. Anchors that are farther away result in less precise accuracy in lateration methods [6]. Consequently, it is important to assign higher weights to the anchor nodes that are in closer proximity to the target node, in order to prioritize the more reliable distance estimates from these nearby locators.

A. *State of the Art Summary*

The study in [8] proposes a lateration method for indoor positioning using BLE beacons, focusing on effective combinatorial beacon selection to improve accuracy and reduce computational cost. The method utilizes a two-stage approach: 1) an initial selection of a subset of beacons based on signal strength and geometric distribution, and 2) a refined selection using a genetic algorithm to optimize the chosen subset further. The paper demonstrates the effectiveness of the proposed method through simulations and experiments, showing improved accuracy compared to traditional lateration methods and reduced computational complexity.

The study in [9] proposes a Weighted Least Squares technique for improved Received Signal Strength (RSS) based localization. The article explores the use of weighted least squares (WLS) techniques to improve the accuracy of indoor positioning based on Received Signal Strength Indicator (RSSI) measurements from BLE beacons (locators). The paper discusses various WLS approaches, including adaptive weighting based on RSSI variance and outlier detection mechanism to mitigate the impact of noisy measurements. Simulations and real-world experiments showcase the effectiveness of WLS techniques in enhancing positioning accuracy compared to conventional least squares methods. However, the existing studies did not explicitly factor in the instantaneous proximity of the target node to the individual anchor nodes, even though this proximity information is readily available as input to the localization algorithm.

Various studies explore recent advancements in indoor localization, emphasizing trilateration as a classic geometric-based model using RSSI data [1], [10], [11], [12], [13]. To improve accuracy, various techniques are presented for Line-of-Sight (LOS) and Non-Line-of-Sight (NLOS) scenarios [14]. Nonlinear stochastic filters, such as Kalman filter (KF) and particle filters, enhance the performance of dynamic localization methods [10], [15]. Various studies delve into different application scenarios, examining measurement data such as RSSI [16], TDOA (Time Difference of Arrival), Time of Flight (ToF), and RTT (round-trip time). Key localization techniques, including RSSI-based fingerprinting [11], employ supervised machine learning methods such as Support Vector Machine (SVM), KNN (K Nearest Neighbors), and CNN (Convolutional Neural

Network) during an offline training/fingerprinting phase [2], [15], [17], [18]. Additionally, unsupervised methods such as isolation forest, k-means, and expectation maximization contribute to enhancing localization accuracy during online testing [10], [19].

This paper presents method for single antenna devices for the simple implementation with Internet of Things (IoT) devices such as BLE enabled devices. Hence, more advanced techniques that employ multiple antennas such as Angle of Arrival (AoA) is out of scope of this paper [15], [17].

In the following section, contributions of this paper are outlined.

B. Contributions of the Paper

The initial estimate of the node's position is obtained using a nonlinear Least Squares (LSQ) lateration method, as described in [20]. Following such initial estimation of the node position through the estimated distances to locators, the novel procedures involve the following further optimizations:

1) Re-calculating the distances from the initial estimate of the node position to each one of the locators,
2) Computing the error vector between these new distance estimates and the initial distance estimates that were used for lateration,
3) Constructing a weighted Jacobian matrix, with weights proportional to the inverse of the estimated distances between the node and each locator,
4) Iteratively updating the node's position using the LM algorithm until a target error is achieved.

The elements of the novel cost function (error vector) are weighted inversely proportional to the estimated distances between the node and the locators. This approach gives more significance to the locators closer to the target node, as their distance estimates are more reliable due to stronger signal levels. The node's initial position estimate is further updated iteratively using the Levenberg-Marquardt (LM) algorithm, which combines the benefits of the Gauss-Newton and gradient descent methods. This is a departure from traditional nonlinear LSQ techniques, as the novel weighted Jacobian matrix with the LM algorithm ensures that the updates on the position estimate prioritize the more accurate distance estimates. This leads to a more accurate position estimate, which is illustrated in the Numerical Results section.

The proposed methods in this paper represent a novel approach that has not been extensively explored in the existing literature. This unique cost function along with weighting scheme is a significant contribution of this work. Additionally, the combination of the weighted Jacobian matrix and the LM algorithm is a unique method. This hybrid approach, which leverages the benefits of both the weighted formulation and the LM optimization technique, marks an important advancement over traditional nonlinear LSQ methods for positioning estimation such as [20]. The weighted Jacobian matrix ensures that the updates on the position estimate prioritize the more accurate nearby distance measurements. By employing these innovative components - the weighted lateration and the LM-based optimization - the proposed algorithm demonstrates superior performance compared to the previously published approaches such as [9] and [19]. The novelty of the presented method lies in its ability to effectively utilize the proximity information of the locators to enhance the overall localization accuracy, particularly in challenging non-line-of-sight (NLOS) scenarios. Additionally, the RSSI samples are curated, with utilization contingent on successful packet reception determined by a Cyclic Redundancy Check (CRC), thereby mitigating RSSI fluctuations induced by fading channel and co-channel interference. Unlike studies in [9] and [19] that considered variance or square magnitude of RSSI samples in their cost function, our proposed method focuses on immediate proximity values to the locators, prioritizing reliability and close-by locators. This results in improved performance not only in LOS but also in NLOS conditions, as opposed to earlier methods that primarily report results for LOS conditions (corresponding to a path-loss exponent of 2).

II. DISTANCE ESTIMATION

This section introduces the distance estimation method, with the subsequent presentation of the channel model.It is important to note that the proposed optimization algorithm is independent of the specific propagation path loss model. The algorithm requires an estimate of the distance to the anchor nodes at its input, without any explicit dependence on the parameters of the path loss model. By utilizing the reciprocal of the distance estimates as weights, the algorithm is able to prioritize the more reliable, nearby anchor nodes and iteratively converge towards a position estimate that minimizes the overall Euclidean distance error, denoted by the target threshold ϵ.

This approach effectively decouples the performance of the algorithm from the accurate knowledge of the propagation model, making it a more generally applicable technique for RSSI-based indoor localization. Of course, any improvement in the initial distance estimations will lead to better performance of localization algorithms.

A. Channel Model

In this section, the channel model is presented for the path-loss model and shadow fading for an indoor environment. Whereas this study can be extended to other types of environments. According to findings in [21], [22], the following path-loss model is assumed:

$$
\begin{aligned}
PL(d) &= PL(d_0) + 10(n_0)\log_{10}(\frac{d}{d_0}) + X(0,\sigma), \\
&\quad \text{for } d \leq d_1, \\
PL(d) &= PL(d_0) + 10(n_0)\log_{10}(\frac{d}{d_0}) + 10(n_1)\log_{10}(\frac{d}{d_1}) \\
&\quad + X(0,\sigma), \text{ for } d > d_1.
\end{aligned} \tag{1}
$$

In 1, PL(d) represents the path-loss in dB at distance d meters between the devices. The term $PL(d_0)$ represents the path loss in dB at the reference distance d_0, n_0 and n_1 are the path loss exponents, d is the distance between the transmitter and receiver, d_0 is the reference distance (where $d_0 < d_1$), and d_1 is the breakpoint where the path loss exponent transitions from n_0 to n_1. The term $X(0, \sigma)$ (in dB) is a random variable that represents the large-scale shadow fading effects, where its mean is zero, and its standard deviation is σ, expressed in decibels. The uncertainty in the channel is represented by its standard deviation σ, which translates into distance estimation errors representing real world scenarios. Larger standard deviations decrease the precision of localization algorithms [6], [23].

III. ITERATIVE WEIGHTED LATERATION METHOD

In this section, we introduce a new iterative approach to improve performance of lateration. We start with using the LM Algorithm, as detailed in [5], [24]. This algorithm is used for nonlinear LSQ curve-fitting problems. This paper takes a pioneering step by customizing this algorithm specifically for the lateration process. In doing so, the algorithm assigns greater importance to locators that are anticipated to be in closer proximity to the End node (EN). This adaptation improves the accuracy and efficiency of the lateration algorithm, marking a significant contribution to the field.

The algorithm continues iterating until a specified convergence threshold is met. It is particularly useful for applications that require accurate localization of a node based on distance measurements from multiple locators, such as indoor positioning systems and wireless sensor networks.

Inputs to the algorithm are as follows:

1) The number of locators, denoted by N_L.
2) The calibrated locator positions, denoted by $(x, y)_L$, which is a vector of $2 \times N_L$ for 2D localization.
3) The initial distance estimates to the EN to be localized, obtained from curated RSSI samples. These initial distances, representing the estimate of the radius from the EN to each locator, are denoted as d, a vector of size $1 \times N_L$.
4) The initial location is determined through lateration employing a non-linear least squares algorithm and is represented as $(\hat{x}, \hat{y})_{\text{init}}$, which is of size 2×1 for 2D localization.

The proposed algorithm improves the accuracy of this initial location estimate iteratively and is described as follows. The output of the algorithm is the new position estimate, or the updated estimate, for the location of the EN. It is denoted by $(\hat{x}, \hat{y})_u$, which is of size 2×1 as well.

The proposed algorithm iteratively finds the parameters to solve for (2).

$$(J_w J_w^T + \lambda I)\delta = J_w \epsilon_w^T, \tag{2}$$

where the right hand side (RHS) of this equation is of size 2x1 for 2D localization. The distance error vector, represented by δ, is specified to belong to the real numbers $\mathbb{R}$ and has dimensions 2×1 for 2D localization. The Left Hand Side (LHS) and RHS of (2) can be expressed using the following formulations:

$$A\delta \equiv B \tag{3}$$

$$A \equiv (J_w J_w^T + \lambda I) \tag{4}$$

$$B \equiv J_w \epsilon_w^T \tag{5}$$

The goal of the iterative algorithm is to minimize the Euclidean norm ($\|\delta\|_2$) of the 2D Cartesian position error vector δ, with units in meters. The Euclidean Norm of a vector v with N elements is defined as: $\|v\|_2 = \sqrt{v_1^2 + v_2^2 + \ldots + v_N^2}$. Let $(x_i, y_i)_L$ represent the known Cartesian location of the Locator i, where $i \in \{1, 2, ..., N_L\}$. The algorithm is initialized as follows: Set maximum number of iterations: $N_{\text{max}} = 100$; Set the coefficient used for weighting:$\beta = 0.1$; Set the coefficient used for regularization: $\lambda = 0.01$; Set the target error (in m): $\epsilon = 10^{-6}$; Initialize the updated location estimate with the initial estimate: $(\hat{x}, \hat{y})_u = (\hat{x}, \hat{y})_{\text{init}}$. Following these initializations, the iterative loop is executed as follows:

for $(k = 1 : N_{\text{max}})\{$

Replicate the updated location estimate to form a $2 \times N_L$ vector:

$$(\hat{x}, \hat{y})_{u,\text{rep}} = \begin{bmatrix} \hat{x}_u & \hat{x}_u & \cdots & \hat{x}_u \\ \hat{y}_u & \hat{y}_u & \cdots & \hat{y}_u \end{bmatrix}; \tag{6}$$

After location estimate update, calculate the RMS distance error to each locator:

$$d_u = \sqrt{\sum_{l=1}^{2} ((x_i, y_i)_L - (\hat{x}_i, \hat{y}_i)_{u,\text{rep}})^2}; \text{ of size } 1 \times N_L \tag{7}$$

Calculate the distance error vector of size $1 \times N_L$:

$$\Delta = d - d_u; \tag{8}$$

Compute the Jacobian matrix (Gradient) of size $2 \times N_L$:

$$J = -\frac{((x, y)_L - (\hat{x}, \hat{y})_{u,\text{rep}})}{d_u}; \tag{9}$$

$$\varepsilon_w = \beta \cdot \Delta; \tag{10}$$

$$J_w = \beta \cdot J; \tag{11}$$

Compute matrix$A$:

$$A = J_w \cdot J_w^T + \lambda \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \tag{12}$$

Compute matrix$B$:

$$\text{B} = J_w \cdot \varepsilon_w^T; \tag{13}$$

Solve for δ :

$$\delta = A^{-1}\text{B}; \tag{14}$$

Update the position estimate by adding the estimated position error:

$$(\hat{x}, \hat{y})_u = (\hat{x}, \hat{y})_u + \delta; \tag{15}$$

Exit the loop when the target position error is met:

$$\text{if } (\|\delta\|_2 < \epsilon)\{ \tag{16}$$

$$\text{break;} \tag{17}$$

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The iterative algorithm presented in this paper, as outlined in (2) to (17), exhibits better performance when contrasted with the lateration algorithms presented in [9] and [20]. This performance improvement will be illustrated in Section IV. This newly introduced algorithm will be referred to as the iw-LM algorithm, where "i" and "w" denote iterative and weighted characteristics, respectively. For clarity, the recently published algorithm in [20] will be designated as the "LSQ algorithm" throughout this paper. A comprehensive description of such traditional lateration algorithm is provided in the following set of equations. When there are multiple locators (with a total of N_L), each with known 2D Cartesian locations, the matrix $\mathbf{A}$ is constructed as follows:

$$\mathbf{A} \equiv 2 \times \begin{bmatrix} (x_1 - x_{N_L}) & (y_1 - y_{N_L}) \\ (x_2 - x_{N_L}) & (y_2 - y_{N_L}) \\ \vdots & \vdots \\ (x_{N_L-1} - x_{N_L}) & (y_{N_L-1} - y_{N_L}) \end{bmatrix} \tag{18}$$

The vector $\mathbf{b}$ is constructed as follows:

$$\mathbf{b} \equiv \begin{bmatrix} d_1^2 - d_{N_L}^2 + x_{N_L}^2 - x_1^2 + y_{N_L}^2 - y_1^2 \\ d_2^2 - d_{N_L}^2 + x_{N_L}^2 - x_2^2 + y_{N_L}^2 - y_2^2 \\ \vdots \\ d_{N_L-1}^2 - d_{N_L}^2 + x_{N_L}^2 - x_{N_L-1}^2 + y_{N_L}^2 - y_{N_L-1}^2 \end{bmatrix} \tag{19}$$

where (x_i, y_i) represents the Cartesian coordinates of the locator $i, i \in \{1, 2, ..., N_L\}$. The vector d stores the estimated distances between the node and each of the N_L locators.

With these definitions, the node position estimate, denoted by $(\hat{x}, \hat{y})$ is determined as follows [20]:

$$(\hat{x}, \hat{y})^T = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}. \tag{20}$$

The computational complexity of the iw-LM algorithm presented in this paper can be expressed as follows for 2D localization in Cartesian space. Given the computations required as described by (2) to (17), the complexity of the algorithm can be represented by $\mathcal{O}(k_{\text{max}} \times N_L)$. Here, $k_{\text{max}} \leq N_{\text{max}}$ represents the number of iterations needed to achieve desired accuracy (ϵ), and N_L denotes the number of locators. In comparison, the nonlinear LSQ algorithm represented by (18), (19) and (20) has a complexity of $\mathcal{O}(N_L - 1)$. The additional factor of k_{max} in the iw-LM algorithm accounts for its iterative nature, leading to a higher overall computational complexity compared to the nonlinear LSQ algorithm. However, the enhanced accuracy provided by the LM algorithm, fast convergence within an average of 4 iterations, as demonstrated in the Numerical Results section, justifies the increased computational cost.

IV. NUMERICAL RESULTS

In this section, the numerical results are presented, which is obtained with the following system settings. In the path-loss model, the following parameters are adopted, tailored to a home environment and applicable to the 2.4 GHz band [22]: $PL_0 = 12.5$ dB, $n_0 = 4.2$, $d_0 = 1$ m, $d_1 = 11$ m, $n_1 = 7.6$, and the standard deviation of log-Normal fading at $\sigma = 3$ dB [25].

To evaluate the performance, a series of Monte-Carlo simulations consisting of 200 independent iterations were conducted. In each iteration, the positions of all the anchor nodes were randomly sampled from a uniform distribution, with the coordinates being statistically independent and identically distributed (i.i.d.) within the boundaries of the simulation area. Each iteration had a duration of 10 seconds, during which 50 distinct end-node positions were sampled once every 200 ms. The mean position estimation error and corresponding cumulative distribution function (CDF) of the errors across all iterations are computed. This statistical analysis provides a comprehensive evaluation of the system's performance, including the characterization of the worst-case scenarios of device placement within the given indoor area. The CDF obtained from Monte-Carlo simulations depicts the probability that the mean estimation error is less than or equal to the value on the x-axis, with the y-axis representing probabilities from 0 to 1. The x-axis value corresponding to a CDF probability of 1 indicates the maximum mean estimation error, potentially arising from anchor orientations with greater distances to the target node. Conversely, a probability approaching zero represents the minimum achievable mean estimation error of the algorithm, potentially arising from anchor orientations with smaller distances to the target node.

The initial estimate of the location of the EN is first estimated assuming the nonlinear LSQ algorithm, which is then smoothed with a moving average filter with window size of 4 past samples [1]. After applying smoothing across multiple samples to mitigate random shadow fading and improve the sensitivity of the BLE nodes, the iw-LM algorithm is then applied. The iw-LM algorithm converged within an average of $k = 4$ iterations per positioning of the locators. The simulation results assuming a LOS environment with $\sigma = 0$ dB shows that the top percentile 95 $\%$ (TP-95) value of position estimation accuracy is 10 cm. This is the positioning accuracy that can be achieved with iw-LM algorithm with 4 locators in a LOS environment and a 20 m border size (400 $m^2 \approx 4,305$ ft^2 area). These values can be important for assessing the system's expected performance and reliability in ideal conditions. In the following figures (Fig. 1 and Fig. 2), the real-world conditions such as N-LOS environment is presented with $\sigma = 3$ dB. Results show the following: The TP-95 positioning accuracy is 2.2 m with only 4 locators covering 400 $m^2 \approx 4,305$ ft^2 area. This accuracy improves to 1.5 m in an NLOS environment with 8 locators. If the locator density is further increased to 8 locators to cover 100 $m^2 \approx 1076.4$ ft^2, (with one locator per 135 ft^2) then sub-meter accuracy can be achieved for 95% of the cases (ref Fig. 3). These findings are summarized in Tab. I.

V. CONCLUSIONS AND FUTURE WORKS

The availability of multiple gateways in a building can be tailored to perform real time localization. Multiple gateways

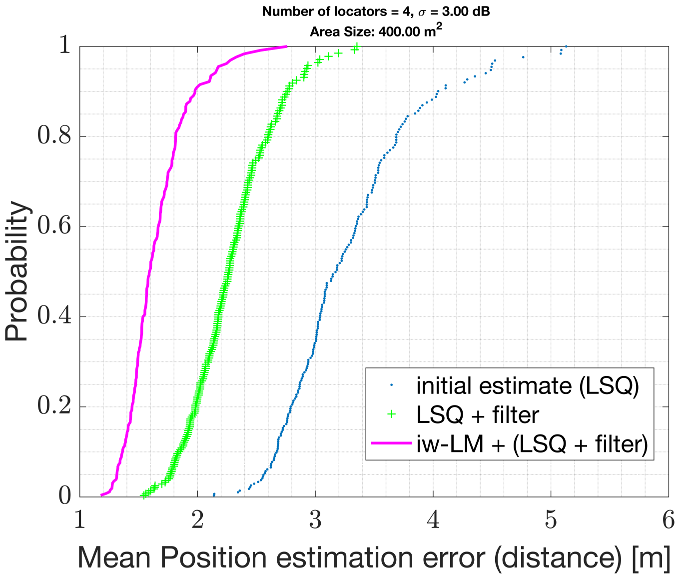

Fig. 1. The CDF of the localization accuracy with $N_L = 4$, Area Size = $20 \times 20\ m^2$.

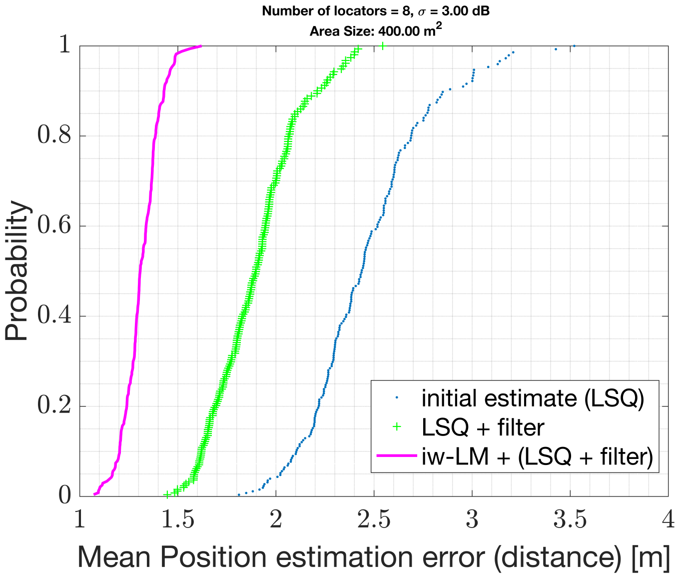

Fig. 2. The CDF of the localization accuracy with $N_L = 8$, Area Size = $20 \times 20\ m^2$.

TABLE I
TP-95 ACCURACY (IN M) VERSUS NUMBER OF LOCATORS IN NLOS

Number of locators	Area Size	Accuracy with iw-LM algorithm	Accuracy with LSQ algorithm
4	$400m^2$	2.2 m	4.5 m
8	$400m^2$	1.5 m	3.1 m
8	$100m^2$	< 1.0 m	2.3 m

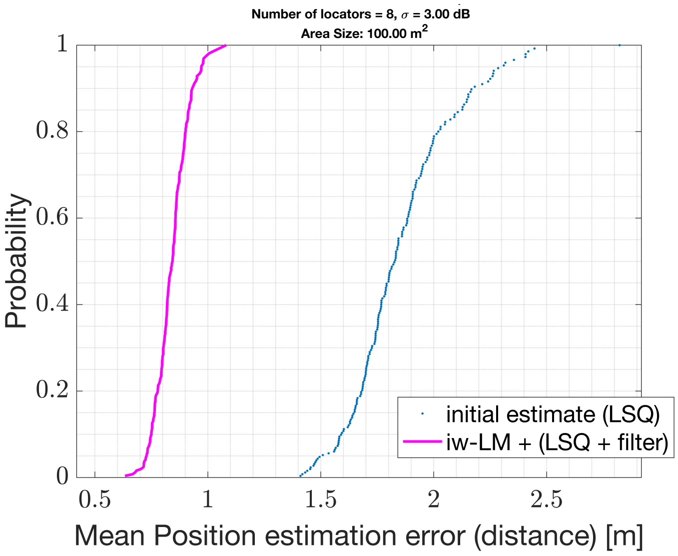

Fig. 3. The CDF of the localization accuracy with $N_L = 8$, Area Size = $10 \times 10\ m^2$.

are essential for ensuring robust and widespread network coverage. These gateways can serve not only as critical access points, but also as locators. Multiple locators expand the network's reach and reliability, especially in larger or more complex environments such as smart cities [21], [26]. Lateration scales well starting with at least four locators. This is needed in order to attain reasonable performance with single antenna devices which lack angle information. Our research demonstrates that increasing the locator density can significantly enhance the accuracy further. In indoor environments affected by multi-path shadow fading effects, a TP-95 localization accuracy of 1.5 meters can be attained when there is one locator for every 538 ft^2 of area. However, halving the locator density (to one locator per 1,076 ft^2) leads to a TP-95 accuracy of 2.2 meters. This emphasizes the critical role played by locator density in achieving precise and reliable localization outcomes.

RSSI-based localization is challenging due to the sensitivity of RSSI measurements to various factors such as interference, multi-path reflections, and attenuation caused by obstacles. These factors introduce significant variability in signal strength, making accurate location estimation difficult. Additionally, the non-linear relationship between RSSI and distance further complicates the localization process, necessitating sophisticated techniques and fingerprinting to achieve precise results. This paper addresses these problems with a new algorithm to do localization with RSSI data curation and iterative optimization. Data curation refers to using only the RSSI samples that pass a CRC check, thereby reducing the impact from interference, noise and fading. The RSSI based localization methods presented in this paper are applicable to any IoT device with single antenna. The performance evaluation is showcased using BLE-1 Mbps signals as a proof of concept, and there is potential for expansion to encompass various lower-bitrate IoT technologies such as 802.15.4 GFSK ([26]) and LoRa ([21]), which can enhance the coverage range.

Future works stemming from this study encompass various avenues. Firstly, the presented 2D lateration algorithm can be extended to do 3D triangulation. The method can also be extended to factor in multiple antenna devices, where available. Our next objective is to develop an improved data curation algorithm that effectively mitigates frequency-selective fading by harnessing the RSSI from multiple channels. Furthermore, we plan to use BLE 5.4 Channel Sounding (CS) with Phase Based Ranging (PBR) mode to improve distance estimations. PBR does not need path-loss model extraction as it relies on ToF estimation which is directly proportional to the distance [27]. The enhanced ranging capabilities of BLE-5.4, particularly the CS with PBR mode, provide a promising venue for practical implementations of the proposed method. The hands-on demonstration with BLE CS enabled devices would offer concrete evidence of the practical benefits of the proximity-guided anchor weighting approach, further validating its advantages over conventional localization methods. Such a demonstration can advance the state-of-the-art in cost-effective and accurate indoor positioning solutions using widely-adopted Bluetooth technology, even with single antenna devices.

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